ANALYSIS OF PORE PRESSURE GENERATION AND DISSIPATION IN COHESIONLESS MATERIALS DURING SEISMIC LOADING

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The earthquake loading of a shallow foundation resting on top of a cohesionless layer creates cyclic variations in the shear force and overturning moment acting on the supporting soil. These loads induce a tendency for volume change which, in turn, depending on the drainage conditions and material permeability, may cause in addition to a cyclic pore pressure variation a progressive pore pressure buildup. The paper develops an efficient and elegant way, based on a multiple time scale analysis, of solving this fully coupled problem. The theoretical solution is implemented in a finite element code and is applied to predict the pore pressure development and dissipation under a bridge pier foundation for which it was essential to limit the pore pressure increase.

Keywords: pore pressure, cohesionless material, earthquake loading, coupled problem.

1. Introduction

Many problems in earthquake engineering are related to the evaluation of pore pressure buildup in cohesionless materials. Without reaching the state where liquefaction is triggered, development of excessive pore pressures beneath a foundation may lead to excessive soil softening or to partial loss of resistance leading to unacceptable foundations settlements and even to bearing capacity failures. A detailed analysis of the development-dissipation of earthquake induced pore pressures requires not only an efficient numerical computer code but also an adequate description of the soil constitutive behaviour. Many constitutive laws are available in the technical literature but they are usually more efficient in describing the non linear shear stress-strain behaviour than the non linear volumetric changes. Therefore the results obtained from those very elaborated solutions are no more pertinent than the soil description from which they are derived. This has led many authors to develop approximate solutions in which the relevant soil parameters, namely the undrained cyclic pore water pressure increase, are fitted to experimental laboratory data [e.g. Booker et *al*, 1976]. However, those solutions lack a rigorous mechanical description and do not take into account the deformability of the soil medium.

In this paper, a theoretical framework, conceptually similar to the method proposed by Booker et *al* [1976], but applicable to more general stress paths and accounting for the soil deformability, is presented. This framework is based on the original work carried out by Dormieux [1989] and applied by Dormieux et *al* [1993] to the stability of the seabed under wave action. The basic idea consists in splitting the time scale into two separate scales associated on the one hand with the cyclic loading (fast time) and on the other hand with the steady pore pressure increase and dissipation (slow time). The equations for both problems are derived and the numerical implementation is presented. The procedure is then illustrated for the case of a bridge foundation for which it was essential to limit the pore pressure increase in the gravel layer located just underneath the foundation.

2. Theoretical Analysis

The problem to be solved, defined in Fig. 1, is that of the pore pressure build-up in a porous gravel layer overlying a compliant impervious clay layer, induced by the seismic loads applied to the rigid foundation. The shallow foundation rests on top of the gravel layer, of limited thickness and lateral extent, encased in the in-situ soils. For the problem under consideration, the in-situ surrounding materials are at least two orders of magnitude less pervious than the gravel layer; therefore, they can be considered as impervious during the loading.

During the seismic loading, the foundation experiences cyclic loads, essentially overturning moment and shear force, which induce a progressive pore pressure build-up and a pore pressure fluctuation around the steady increase (Fig. 2). The cyclic pore pressure fluctuation is related to changes in the mean total stress, whereas the progressive build-up is related to the tendency for volume changes in the material. Over one period of loading T_L, the permanent pore pressure build-up is equal to ΔA (Fig. 2).

This pore pressure build-up is counterbalanced by the pore pressure dissipation taking place in the gravel layer from underneath the foundation towards the foundation edges. Whichever the fastest of the two phenomena (pore pressure build-up or pore pressure dissipation) will govern the residual pore pressure at any time.

The solution to the given problem, which is the residual pore pressure at any point and at any time, is based on the theoretical concepts presented in [Dormieux, 1989 and Dormieux et *al*, 1993]. It is shown hereafter that the solution can be obtained from the resolution of two distinct problems:

- a linear elastic problem for an incompressible medium
- a nonlinear thermo-poro-elastic problem with a source term.

The solution is derived by splitting the global problem into two sub-problems according to the time scales involved. The first time scale T_L is related to the cyclic loads (Fig. 2); the second one is related to drainage and is defined by a characteristic time

$$T_c = \frac{R^2}{C_m} \tag{2.1}$$

where R is half the foundation width and C_m is the diffusion coefficient:

$$C_m = (\lambda + 2G)\frac{k}{\gamma_w} \tag{2.2}$$

In which λ and G are the Lame's coefficients, k the soil permeability and γ_w the water unit weight.

Typically, in most common situations both time scales are well separated, T_L being of the order of a second and T_c of the order of many seconds, and advantage can be taken of this property to solve both problems in an uncoupled way. We introduce the parameter α

$$\alpha = \frac{T_L}{T_c} << 1 \tag{2.3}$$

and we define by A($\underline{x}$,t) any given quantity of the problem. $\underline{x}$ defines the position in space and will be omitted in the following but it must be understood that all the quantities referred to depend on $\underline{x}$. In relation with the imposed loading which is assumed periodic with period T_L, it is reasonable to assume that A is a quasi periodic function of time, oscillating with the period T_L around a mean value (Fig. 2). The idea of using a double time scale is equivalent to expressing the quantity A as a function of two dimensionless variables: the fast variable $\tau = t/T_L$ and the slow variable $\theta = t/T_c$. Therefore we let $A(t) = A'(\tau, \theta)$. Intuitively, the variable τ must take into account the rapid variations of the quantity A within the loading cycle. The variable θ expresses the drift of this quantity on the scale of the consolidation phenomenon. Therefore, assuming that A' is a periodic function of period 1 with respect to τ, the drift ΔA of A within a cycle can be entirely related to the variable θ:

$$\Delta A = A(t+T_L) - A(t) = A'(\tau, \theta+\alpha) - A'(\tau, \theta) = \alpha \frac{\partial A'}{\partial \theta} \tag{2.4}$$

in which use is made of the relationship $A(t+T_L) = A'(\tau+1, \theta+\alpha) = A'(\tau, \theta+\alpha)$.

As postulated in Dormieux et *al* [1993], the fluctuations of A can be expanded as functions of increasing powers of the parameter α:

$$A(t) = \sum_{n=0}^{\infty} \alpha^n A'^{(n)}(\tau,\theta) \tag{2.5}$$

in which $A'^{(i)}$ and their derivatives with respect to τ and θ are of the order of $\alpha^{(i)}$ and periodic, of period 1, with respect to τ.

2.1. *Boundary conditions*

They are defined for the global problem in Fig. 1:

- at $z = 0$ $\qquad \underline{\xi} = \underline{0}$ (2.6)
- on (P')

$$\underline{\underline{\sigma}} \cdot \underline{e}_z = -\gamma_w \, d \, \underline{e}_z \; ; \; p = \gamma_w \, d \tag{2.7}$$

with:
$\underline{\xi}$ displacement
p total fluid pressure
$\underline{\underline{\sigma}}$ total stress
d depth of water

- on (P),

the cyclic loads applied to the foundation induce a displacement $\underline{\xi}$ which is assumed to be a periodic function of period T_L:

$$\underline{\xi} = \underline{\xi}^d\left(\underline{x}, \frac{t}{T_L}\right) \; ; \; \frac{\partial p}{\partial z} = -\gamma_w \tag{2.8}$$

where $\underline{x}$ is the position vector and the foundation is an impervious boundary.

- $z = H$,

in addition to the continuity of displacement and stress vector, the fluid flow across the boundary is zero:

$$\frac{\partial p}{\partial z} = -\gamma_w \tag{2.9}$$

2.2. *Constitutive Model*

We assume that the natural ground can be modeled with a one-phase model (undrained condition) with the appropriate constitutive behavior. The gravel layer is modeled as a porous medium with the following constitutive law:

$$\underline{\underline{\varepsilon}} = \underset{\sim 0}{A} : \underline{\underline{\sigma}}' + \frac{1}{3}\,\varepsilon_v^p\,\underline{\underline{1}} \tag{2.10}$$

where $\underline{\underline{\sigma}}'$ denotes the effective stress tensor $(= \underline{\underline{\sigma}} + p\,\underline{\underline{1}})$, ε_v^p the plastic volumetric strain and $\underset{\sim 0}{A}$ the effective tensor of elasticity. For an isotropic medium:

$$\underset{\sim 0}{A} : \underline{\underline{\sigma}}' = \frac{1+\nu_0}{E_0}\,\underline{\underline{\sigma}}' - \frac{\nu_0}{E_0}\,\mathrm{tr}\,\underline{\underline{\sigma}}'\,\underline{\underline{1}} \tag{2.11}$$

In the rest of the paper it is assumed that the volumetric behavior is exclusively contractive, i.e.:

$$\dot{\varepsilon}_v^p \leq 0 \tag{2.12}$$

The proposed modeling does not account for the deviatoric plastic strain since emphasis is placed on the excess pore pressure due to contractance.

2.3. *Problem I: Fast variables*

The problem, which is referred to as problem I, has a time scale related to the period of the cyclic loads. It gives the solution for changes associated with small time increments (changes occurring within one period); these changes are assumed to be periodic with period T_L (fluctuations around the progressive pore pressure build-up).

At the order –1 in α, taking into account Eq. (A.4) from appendix A, the constitutive equation (2.10) takes the form:

$$\frac{\partial}{\partial \tau}\left(\underline{\underline{\varepsilon}}^{(0)}\right) = \underset{\sim 0}{A} : \frac{\partial}{\partial \tau}\left(\underline{\underline{\sigma}}'^{(0)}\right) \tag{2.13}$$

with the incompressibility condition:

$$\frac{\partial}{\partial \tau}\left(\mathrm{tr}\,\underline{\underline{\varepsilon}}^{(0)}\right) = 0 \tag{2.14}$$

The equations for the problem in the fast variable $\tau = t / T_L$ are therefore given by:

(i) the constitutive equations (2.9) and (2.10)

(ii) the boundary conditions

$$\begin{aligned}
&\text{- at } z = 0 \quad \frac{\partial}{\partial \tau}\left(\underline{\underline{\xi}}^{(0)}\right) = 0 \\
&\text{- on (P')} \quad \frac{\partial}{\partial \tau}\left(\underline{\underline{\sigma}}^{(0)}\right).\underline{e}_z = 0 \\
&\text{- on (P)} \quad \frac{\partial}{\partial \tau}\left(\underline{\underline{\xi}}^{(0)}\right) = \frac{\partial}{\partial \tau}\left(\underline{\underline{\xi}}^{d}\right)
\end{aligned} \tag{2.15}$$

(iii) the balance law

$$\mathrm{div}\,\frac{\partial}{\partial \tau}\left(\underline{\underline{\sigma}}^{(0)}\right) = 0 \tag{2.16}$$

Under this form the problem, in terms of $\frac{\partial}{\partial \tau}\left(\underline{\underline{\xi}}^{(0)}\right)$ and $\frac{\partial}{\partial \tau}\left(\underline{\underline{\sigma}}^{(0)}\right)$, is a problem of elasticity for an incompressible, one phase medium. The excess pore pressure $p^{(0)}$ is the solution of Eqs. (2.13) and (2.14), once this problem solved:

$$\frac{\partial p^{(0)}}{\partial \tau} = -\frac{1}{3}\frac{\partial}{\partial \tau}\left(\mathrm{tr}\,\underline{\underline{\sigma}}^{(0)}\right) \tag{2.17}$$

In fact, the excess pore pressure $\frac{\partial p^{(0)}}{\partial \tau}$ is the Lagrange multiplier of the incompressibility constraint $\frac{\partial}{\partial \tau}\left(\mathrm{tr}\,\underline{\underline{\varepsilon}}^{(0)}\right)=0$.

The main result of this problem lies in the fact that $\frac{\partial}{\partial \tau}(A)$ only depends on $\underline{x}$ and τ (independent of θ !) for all mechanical variables (because of the assumption that $\underline{\xi}^{d}$ is a function of τ alone). Therefore:

$$A(\tau , \theta) = A_\tau (\tau) + A_\theta (\theta) \tag{2.18}$$

where A_τ is the fluctuation (with an average value equal to 0 over a period) and A_θ the drift (i.e. the mean) which appear uncoupled in Eq. (2.18).

In particular, the deviator stress $\underline{\underline{s}}$ writes, according to Eq. (2.18):

$$\underline{\underline{s}}\,(\tau,\theta) = \underline{\underline{s}}_\tau\,(\tau) + \underline{\underline{s}}_\theta\,(\tau) \tag{2.19}$$

The amplitude D of variations in $\underline{\underline{s}}_\tau$ (measured, for instance, by $J_2=\left(\frac{1}{2} s^\tau_{ij}\, s^\tau_{ij}\right)^{1/2}$) constitutes the solution to problem I and obviously governs the contractance. $\underline{\underline{s}}_\theta$, which represents the mean deviator stress and is the solution to problem II, can also affect the contractance.

2.4. *Problem II: Slow variable or average quantities*

Attention is turn to the determination of the quantities $A_\theta(\theta)$ in Eq. (2.18). The diffusion equation at the order 0 in α, Eq. (B.6) in appendix B, can be integrated with respect to τ over a period T_L: because of the single periodicity of $\underline{\underline{\varepsilon}}^{(1)}$ with respect to τ:

$$\frac{\partial}{\partial \theta}\,\mathrm{tr}\left(\underline{\underline{\varepsilon}}^{(0)}_\theta\right) = \Delta'\left(p'^{(0)}_\theta\right) \tag{2.20}$$

In dimensionless variables (slow time variable), the structure of the ordinary diffusion equation (B.1) is retrieved with the drift quantities as variables.
From the condition $\mathrm{div}\,\underline{\underline{\sigma}}^{(0)}=0$ and from the fact that the solution $\underline{\underline{\sigma}}^{(0)}_\tau$ of problem I also satisfies $\mathrm{div}\left(\frac{\partial}{\partial \tau}\,\underline{\underline{\sigma}}^{(0)}_\tau\right)=0$, it can be shown that $\mathrm{div}\,(\underline{\underline{\sigma}}^{(0)}_\theta)=0$.

The equations of the problem for the average quantities are therefore:

(i) boundary conditions:

- at $z = 0$ $$\underline{\underline{\xi}}_\theta^{(0)} = \underline{0} \tag{2.21}$$

- at $z = h$ on (P') $$\underline{\underline{\sigma}}_\theta^{(0)} \cdot \underline{e}_z = -q\,\underline{e}_z \;\; ; \;\; p_\theta^{(0)} = q \tag{2.22}$$

with $q = \gamma_w d$

on (P) $$\underline{\xi}_\theta^{(0)} = \underline{0} \;\; ; \;\; \frac{\partial}{\partial z} p_\theta^{(0)} = \gamma_w \tag{2.23}$$

- at $z = H$ $$\frac{\partial}{\partial z} p_\theta^{(0)} = \gamma_w \tag{2.24}$$

(ii) balance law:

$$\operatorname{div} \underline{\underline{\sigma}}_\theta^{0} = \underline{0} \tag{2.25}$$

(iii) constitutive law:

$$\underline{\underline{\varepsilon}}_\theta^{0} = \underset{\sim}{A}_0 : \underline{\underline{\sigma}}'^{(0)}_\theta + \frac{1}{3}\varepsilon_v^p(\theta)\,\underline{\underline{1}} \tag{2.26}$$

(iv) diffusion equation:

$$\frac{\partial}{\partial \theta}\left(\operatorname{tr} \underline{\underline{\varepsilon}}_\theta^{0}\right) = \Delta' p_\theta'^{(0)} \tag{2.27}$$

The load is therefore (in addition to q) defined by $\varepsilon_v^p(\theta)$ given by the empirical law of contractance presented in paragraph 3.0; it is a function of D solution to problem I. The solution to problem II reduces to the solution of a simple thermo-poro-elastic problem for the quantities $\varepsilon_v^p(\underline{x}, t)$ and $p(\underline{x}, t)$.

3. Model for Pore Pressure Generation

The source term ε_v^p is given by:

$$\frac{\partial \varepsilon_v^p}{\partial\theta} = \frac{1}{\alpha}\, f\left[\frac{D}{\sigma'_m}, \varepsilon_v^p(\theta)\right] = \frac{b_1}{\alpha}\left(\frac{D}{\sigma'_m}\right)^{b_2} \exp(b_3\,\varepsilon_v^p) \tag{3.1}$$

where:

$D \quad = J_2 = \frac{1}{2}\left(s_{ij}^\tau s_{ij}^\tau\right)^{1/2}$

σ'_m mean effective stress

$\alpha \quad = \dfrac{T_L}{T_c}$

$\theta \quad = \dfrac{t}{T_c}$

b_1, b_2, b_3 material parameters

The exponential term in Eq. (3.1) accounts for volumetric hardening. Although its functional form is different, Eq. (3.1) expresses the dependency of the volumetric strain increment on the same fundamental parameters as other equations, such as the one used in the well-known Martin et *al* model [1975]:

. amplitude of shear stress (or shear strain)
. accumulated volumetric strain
. present state of stresses.

In the absence of specific laboratory data, the parameters entering Eq. (3.1) can be determined from the experimental data presented by Banerjee et *al* [1979]. Experimental data are provided for the bulk modulus of coarse-grained materials and for the dynamic pore water pressure increase in cyclic triaxial tests. These data are presented in Figs. 3 and 4.

From Fig. 3, the coefficient of volume compressibility should range from 0.006 to 0.03 MPa^{-1}, or equivalently, the bulk modulus should range from 30 to 160 MPa; values for Oroville gravel are equal to 150 MPa. Fig. 4 shows that the rate of pore pressure increase in coarse-grained material is higher than in sands (results from [Lee and Albaisa, 1974]).

To check the validity of Eq. (3.1) and to determine the appropriate material parameters, numerical simulations of cyclic undrained triaxial tests of Oroville gravel have been performed. The pore pressure is computed in the following way:

$$\dot{\varepsilon}_v^p + \dot{\varepsilon}_v^e = \frac{n}{B_w}\dot{u} \tag{3.2}$$

which states that the total volume change (plastic ε_v^p plus elastic ε_v^e) is equal to the pore water compressibility (n = porosity)

$$\dot{\varepsilon}_v^e = \frac{\dot{\sigma}'_m}{B_s} = \frac{\dot{\sigma}_m - \dot{u}}{B_s} \tag{3.3}$$

From Eqs. (3.2) and (3.3), it comes:

$$\dot{u} = \frac{\dot{\sigma}_m / B_s + \dot{\varepsilon}_v^p}{\frac{n}{B_w} + \frac{1}{B_s}} \tag{3.4}$$

In a cyclic triaxial test, or in a simple shear test, for one cycle $\dot{\sigma}_m = 0$. In addition, the bulk modulus of water $B_w \approx 2\,000$ MPa is an order of magnitude higher than that of the soil skeleton B_s. Therefore, Eq. (3.4) simplifies to:

$$\Delta u = B_s\, \Delta\varepsilon_v^p \tag{3.5}$$

where Δu and $\Delta\varepsilon_v^p$ are the pore pressure increment and volumetric strain at the end of a cycle; $\Delta\varepsilon_v^p = T_L\,\dot{\varepsilon}_v^p$, where $\dot{\varepsilon}_v^p$ is given by Eq. (3.1).

For a conventional cyclic triaxial test, the cyclic shear stress is equal to $\sigma_d/2$ where σ_d is the cyclic deviator stress and $J_2 = \sigma_d^2 / 3$; therefore, $D = \sigma_d / \sqrt{3}$.

The simulation of the pore pressure build-up for two cyclic stress ratios ($\tau/\sigma = \sigma_d / 2\,\sigma_3 = 0.4$ and 0.6) is shown in fig. 5 with the experimental data; they show that with the appropriate choice of the material parameters, Eq. (3.1) adequately represents the rate of pore pressure build-up.

These properties with a skeleton bulk modulus of 150 MPa are used in the following calculations. The other parameter required for the analysis is the gravel permeability, which has been varied between 10^{-4}m/s and 10^{-2} m/s.

4. Numerical implementation

The equations are solved by using the finite element method. The soil foundation spatial domain is subdivided into two sub-domains corresponding to:

- subdomain 1: porous gravel layer
- subdomain 2: impervious clay layer

Within the impervious clay layer, the dependent variables are the solid displacements, and the usual solid equations are solved. The incompressibility constraint is imposed by the penalty method by assigning a large bulk modulus to the fluid phase. Reduced selective integration and/or a mean dilatational formulation (viz., $\overline{B}$ methods) are used to avoid locking of the elements (Hughes, 1980).

Within the porous gravel layer, the dependent variables are the solid skeleton displacements and the pore-water pressures. The governing equations are coupled. Each component field is spatially discretized into a finite element method, and naturally gives rise to what is termed a "mixed method." For a convenience standpoint, in our implementation we use equal-order interpolations for both the solid displacement and the fluid pressure (typically C° interpolations). Such an implementation is known in general to fail to satisfy the Babuska-Brezzi condition (Babuska, 1971; Brezzi, 1974), and can lead to violently oscillating pressure field especially near the incompressible limit case (i.e., when very small or no fluid diffusion can take place). The correct for that deficiency, we use a Petrov-Galerkin stabilization formulation (Hughes et al, 1986) to enhance stability without upsetting consistency. Such a formulation has been shown to be stable and convergent for rather general C° combinations of displacement and pressure.

The resulting first-order coupled semi-discrete finite element matrix equations are integrated in time by using the generalized trapezoidal family of finite difference time stepping algorithms. Typically, we use a backward Euler scheme to maximize high-frequency dissipation. This results in a non-linear algebraic system of equations to be solved at each time step. The solution of this system is obtained by using an iterative strategy which is implemented by means of a predictor-multicorrector scheme applied at each time step. In this method, a series of corrected solutions are computed starting from an initial or predicted solution for the time step. Each corrected solution is used in the following iteration to compute the next corrected solution. The procedure continues until a specified solution convergence has been obtained. There are several ways of implementing the recursive relationships that take the solution from step n to step $n + 1$. In our implementation we always chose to use the vector of nodal quantities with the highest time derivatives as the vector of primary unknown variables. The resulting system of equations is solved by performing a linearization via a truncated Taylor's series expansion, and results in the following coupled linear system of equations to be solved at each iteration:

$$\begin{bmatrix} \alpha \Delta t \mathbf{K} & -\alpha\ \Delta t \mathbf{L} \\ \mathbf{L}^T - \ell & \mathbf{M} + \alpha \Delta t \mathbf{H} \end{bmatrix} \begin{bmatrix} \Delta \dot{\mathbf{u}}_{n+1}^{(i)} \\ \Delta \dot{\mathbf{p}}_{n+1}^{(i)} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_{u,n+1}^{(i)} \\ \mathbf{r}_{p,n+1}^{(i)} \end{bmatrix} \tag{4.1}$$

where the superscript index i is used to index the non-linear iterations; α is the time-stepping parameter (typically, $\alpha = 1$); $\mathbf{r}_u$ and $\mathbf{r}_p$ denote the residuals which are evaluated from initial and known values from the previous iterate; $\dot{\mathbf{u}}$ is the vector of unknown nodal solid velocities; $\dot{\mathbf{p}}$ is the vector of unknown nodal fluid pressure time derivatives; **K** is the solid stiffness matrix; **L** is the gradient matrix; $\mathbf{L}^T$ is the divergence matrix; ℓ is the Petrov-Galerkin "consistency" matrix; **M** is a combination of the Petrov-Galerkin "stabilization" matrix and the mass matrix; **H** is the diffusion (permeability) matrix.

At the core of the combined simultaneous solution procedure is the solution of the set of linear equations Eq. 4.1. If the full matrix system is to be used, the assembly and factorisation of the non-symmetric coefficient matrix is found to pose enormous computation demands, especially for three-dimensional problems. It also requires that a special software module be developed to combine the field equations, hampering the direct use of discipline-oriented software modules for each field equation. To preserve software modularity and to avoid having to operate with the full nonsymmetric matrix equation, we solve Eq. 4.1 by using a partitioned iterative conjugate gradient procedure (Prevost, 1997). The procedure has been shown to be unconditionally stable and does not require that the full system of coupled equations be assembled nor that the coupling matrices $\mathbf{L}$, $\mathbf{L}^T$ and ℓ ever be formed or assembled. This procedure results in substantial computational savings.

The implementation and synchronization of the equations are achieved by defining solution staggers as follows:

- solid stagger 1: solves solid equation in the impervious clay layer
- solid stagger 2: solves solid equation in the porous gravel layer
- pressure stagger: solves diffusion equation in the porous gravel layer.

The calculations then proceed as follows:

- Phase 1: initialization: computation of $J_2(\mathbf{x}, t=0)$ in the gravel layer when the full amplitude of the cyclic force and moment are applied to the foundation system. In this case, the material in the gravel layer is assumed to be linear-elastic and no diffusion is allowed to take place (viz., undrained conditions).
- Phase 2: external loads are applied due to the self weight of the foundation and soil materials. In this case, full diffusion is allowed in the gravel layer and no excess pore-water pressures are allowed to take place (viz., drained conditions).
- Phase 3: generation (Eqs. 3.1 and 3.5) and diffusion of excess pore-water pressures in the gravel layer due to $J_2(\mathbf{x}, 0)$. In this case, the full system of coupled equations (solid stagger 2 and pressure stagger) are used to compute the time dependent solid displacements (effective stresses) and of pore-fluid pressures in the gravel layer.

5. Application

The method of solution developed in the preceding paragraphs has been applied to evaluate the earthquake induced pore pressure buildup in the soil underneath the piers foundations of a bridge. The project under consideration is the Rion-Antirion bridge under construction. After a brief presentation of the project, the analyses of the pore pressure buildup and dissipation in the gravel backfill placed on top of the natural ground are presented and it is shown how they have been used to define the material specifications for the backfill.

5.1. *Description of the project*

The Rion-Antirion bridge project is a BOT contract granted by the Greek Government to a consortium led by the French company Dumez-GTM. It is located in Greece, near Patras, and will constitute a fixed link between the Peloponese and the Continent. The solution adopted for this bridge is a multiple spans cable stayed bridge with four main piers; the three central spans are 560 m long each and are extended by two adjacent spans (one on each side) 265 m long. The total length of the bridge, with the approach viaducts, is approximately 2.5 kilometres.

The bridge is located in a highly seismic area and has to be designed for severe environmental conditions [Teyssandier et *al*, 2000]:

(i) the foundation soils consist in soft alluvial deposits (silty clays, clayey silts) extending to depth in excess of 500 metres. The mechanical characteristics of these layers are rather poor with undrained shear strengths increasing slowly with depth from approximately 30-50 kPa at the sea bed level to 80-100 kPa at 50 m depth; the shear wave velocities are also small, increasing from 100-150 m/s at the ground surface to 350-400 m/s at 100 m depth;

(ii) the water depth in the middle of the strait reaches 65 m;

(iii) the seismic design motion corresponding to a 2 000 years return period earthquake has a peak ground acceleration (at the sea bed level) of 0.48 g and a 5% damped response spectrum with a plateau at 1.2 g extending from 0.2 s to 1.1 s; at 2 s the spectral acceleration is equal to 0.62 g;

(iv) the bridge must accommodate a 2 m differential tectonic displacement in any direction and between any two piers.

In order to accommodate those conditions and to carry the large earthquake forces brought to the foundation (shear force of the order of 500 MN and overturning moment of the order of 18 000 MNm for a vertical buoyant pier weight of 750 MN), the foundation design concept which was finally adopted consists of a gravity caisson (90 m in diameter at the sea bed level) resting on top the reinforced natural ground [Combault et *al*, 2000]. The ground reinforcement is composed of steel tubular pipes, 2 m in diameter, 20 mm thick, 25 to 30 m long driven at a grid of 7 m x 7 m below and outside the foundation over a circular area of 13 300 m^2. The total number of inclusions under each foundation is therefore of the order of 270. In addition, the safety of the foundation is greatly enhanced by interposing a gravel bed layer, 2.8 m thick, on top of the inclusions just below the foundation raft (fig. 6). This concept (inclusions plus gravel layer) enforces a capacity design philosophy in the foundation design, [Pecker, 1998]. The gravel layer is equivalent to the "plastic hinge" where inelastic deformation and dissipation take place and the "overstrength" is provided by the ground reinforcement which prevents the development of deep seated failure mechanisms involving rotational failure modes of the foundation. If the design seismic forces were exceeded, the "failure" mode would be pure sliding at the gravel-foundation interface; this "failure mechanism" can be accommodated by the bridge, which is designed for much larger tectonic displacements than the permanent seismically induced ones.

In order for the gravel layer to act as a fuse at a given shear force level, it is essential that the mechanical characteristics of the gravel itself and of the soil-foundation interface be well controlled. This can only be achieved if the amount of pore pressure buildup remains insignificant in the gravel layer. Development of pore pressures in the gravel would soften the layer, decrease its available shear strength and, more importantly, its amount would be highly unpredictable making the fuse concept very uncertain. It therefore appears essential to make a reliable prediction of the pore pressure buildup in the gravel layer in order to limit it to a minimum; this is achieved with the parametric numerical analyses presented in paragraph 5.2. which, at the end, will guide the choice of the required material permeability and subsequently define the material gradation.

5.2. *Foundation Loading*

The loads applied to the foundation are obtained from an independent dynamic non linear finite element analyses in which all soil strata (gravel bed and natural ground) are treated as non linear one-phase materials. These analyses are performed with the computer code DYNAFLOW [Prévost, 1999]. Typical maximum values are equal to:

(i) horizontal shear force: F = 500 MN

(ii) overturning moment: M = 15 000 MNm to 18 000 MNm depending on the analysed foundation.

Due to the possibility of sliding at the soil-foundation interface, the maximum force cannot exceed $N \tan\phi = 750 \tan 35° = 525$ MN. Therefore, F = 500 MN can be taken as a typical maximum force. For the overturning moment, the sensitivity of the results (pore pressure development) to the exact maximum value of M is far less critical since the contribution of M is mainly to induce changes in the mean stress around the foundation edges and not in the shear stress.

Following the cumulative damage approach used in liquefaction analyses, the time histories of loads at the foundation level are represented by a series of uniform cycles. The amplitude of the cycles is taken equal to 65% of the maximum load amplitude. The number of cycles is taken equal to 15, which is consistent with the earthquake magnitude (6.8 to 7.0); this number of cycles corresponds to a total duration of 15 seconds for the strong phase since the period is of the order of 1.0 second as evidenced from a Fourier transform of the shear force time history at the foundation level (fig. 7). Therefore, the "shear stress" D, solution to problem I, is computed for the following loads applied to the foundation:

. F = 325 MN or 4.0 MN/mℓ in the 2D model
. M = 9 850 MN.m or 120 MN.m/mℓ in the 2D model.

The finite element model used for the calculations of the pore pressure distribution is shown in fig. 8. It is used to compute the initial mean effective stress under the pier weight and the shear stress as a function of position $\underline{x}$ in the gravel layer. The spatial distribution of the shear stress D in the gravel layer is presented in fig. 9. The data of fig. 9, and the equivalent ones giving the mean effective stress (not presented), are used as input data in the plastic volumetric strain Eq. 3.1; they constitute the solution to problem I.

5.3. *Pore Pressure Development*

The development of the pore pressure in the gravel layer (solution to problem II) has been computed as a function of time for three different values of the gravel permeability: 10^{-4}, 10^{-3} and 10^{-2} m/s.

Time histories of the pore pressure build-up at points underneath the foundation are shown in figs. 10 to 12 for the three analysed permeabilities. These points are located under the edge of the foundation, 13 m off the centreline and on the centreline, at 0.75 m (elevation of inclusions heads) below the foundation.

For the smallest permeability, except under the very edges of the foundation, the pore pressure increases steadily during the loading to reach values of the order of 30 kPa after 15 seconds. No dissipation takes place under the foundation. For the intermediate permeability value, the pore pressure under the foundation increases until 4 seconds and then levels off, the rate of increase being compensated by dissipation towards the free draining surface. The pore pressure below the foundation reaches a value of 25 kPa. Near the edges, the pore pressure rises rapidly in one second up to 6 kPa and then drops down quickly. For the highest permeability, everywhere under the foundation, the pore pressures buildup to a maximum value of 16 kPa in 1 second, then drops down to values less than 5 kPa in less than 8 seconds. After 2 seconds, the average excess pore pressure is less than 10 kPa under the foundation.

Based on these results, it appears that a ballast layer with permeability higher or equal to 10^{-2} m/s is able to maintain an almost drained condition during the seismic loading despite the long drainage path from the foundation centre to the edges (45 m). The maximum excess pore pressure does not exceed 15 kPa, which represents approximately 10% of the vertical effective stress brought by the foundation, is not sustained during the loading, and dissipates at a fast rate. The critical condition mentioned above, e.g. the ballast layer must remain drained during the earthquake, is therefore satisfied: an effective stress condition prevails in the ballast layer and the relevant strength parameters (effective friction angle) can be reliably assessed. Underwater screening of a dredged river bed gravel close to the site gives rise to a coarse granular clean material with a D_{10} larger than 10mm (less than 10% by weight smaller than 10mm) and a uniform gradation to prevent segregation during placement; its measured permeability (0.5 m/s) is higher than the target one. To avoid pollution in time of the gravel bed layer due to fine migration from the bottom, the layer is protected by a filter layer interbedded in between the natural ground and the gravel layer; pollution from the surface is not a concern because careful examination of the sedimentation conditions at the sea bottom shows that this phenomenon is almost inexistent and moreover the surface of the gravel layer outside the foundation is protected with a scour protection.

6. Summary and Conclusions

An efficient scheme has been developed for solving the problem of progressive buildup and dissipation of pore pressures in cohesionless soils during earthquakes. This method is based on the idea of using two different time scales depending on the problem to be solved: a small time scale is used for determining the rapid fluctuations of the pore pressure within one cycle of loading; a larger time scale is used for determining the steady increase in pore pressure. In that format, the global problem is split into two sub-problems: an elastic problem for an incompressible medium and a thermo-poro-elastic problem with a source term. This scheme has been implemented in the finite element computer code DYNAFLOW using a staggered strategy for the numerical solution. In the present paper the first problem is used to determine the amplitude of the generalized shear stress (second invariant of the deviatoric stress tensor) at any location within the cohesionless layer for a surface foundation loaded by a time varying shear force and overturning moment; solution to the second problem, based on the solution to the first problem and on an experimentally based relationship for the pore pressure increase under undrained conditions, yields the steady pore pressure increase.

The method has been used to predict the pore pressure likely to develop in a gravel bed layer placed under the piers foundations of the Rion-Antirion bridge presently under construction. For that project it was essential to limit the pore pressure increase to the minimum conceivable value, regardless of the long drainage paths, in order for the gravel layer to fully achieve its role of fuse under seismic loading. In that way a capacity design philosophy could be efficiently implemented for the design of the foundations. Results of the analyses were further utilized to define the material specifications in terms of minimum required permeability, or equivalently of grain size gradation.

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Appendix A: Derivation with respect to time.

The chain rule for derivation implies that:

$$\frac{\partial}{\partial t} = \frac{1}{T_c}\frac{\partial}{\partial \theta} + \frac{1}{T_L}\frac{\partial}{\partial \tau} = \frac{1}{T_c}\left(\frac{\partial}{\partial \theta} + \frac{1}{\alpha}\frac{\partial}{\partial \tau}\right) \tag{A.1}$$

Under these conditions, the assumption $\dot{\varepsilon}_v^p < 0$ gives:

$$\frac{\partial}{\partial \theta}\left(\varepsilon_v^p\right) + \frac{1}{\alpha}\frac{\partial}{\partial t}\left(\varepsilon_v^p\right) \leq 0 \tag{A.2}$$

e.g., developing ε_v^p in Taylor's series $\quad \varepsilon_v^p = \varepsilon_v^{p(0)} + \alpha\,\varepsilon_v^{p(1)} + \ldots$

$$\frac{1}{\alpha}\frac{\partial}{\partial \tau}\left(\varepsilon_v^{p(0)}\right) + \frac{\partial}{\partial \theta}\left(\varepsilon_v^{p(0)}\right) + \frac{\partial}{\partial \tau}\varepsilon_v^{p(1)} + \ldots \leq 0 \tag{A.3}$$

To the order - 1 (term in $1/\alpha$), we thus get $\frac{\partial}{\partial \tau}\left(\varepsilon_v^{p(0)}\right) \leq 0$

Using the fact that $\varepsilon_v^{p(0)}$ is periodic with respect to τ, it follows that:

$$\frac{\partial}{\partial \tau}\left(\varepsilon_v^{p(0)}\right) = 0 \tag{A.4}$$

Appendix B: Diffusion equation.

The diffusion equation for an incompressible fluid and solid writes:

$$\mathrm{tr}\,\dot{\underline{\underline{\varepsilon}}} = k\,\Delta\,p \tag{B.1}$$

Defining $\Delta = \frac{1}{R^2}\Delta'$ and $p = (\lambda_0 + 2\mu)\,p'$

It follows: $$k\,\Delta\,p = \frac{1}{R^2}\,k\,(\lambda_0 + 2\mu)\,\Delta'\,p'$$

Or: $$k\,\Delta p = \frac{C_m}{R^2}\Delta' p' = \frac{1}{T_c}\left(\Delta' p'^{(0)} + \alpha\, p'^{(1)} + \dots\right) \tag{B.2}$$

Taking into account Eq. (A.1),

$$\frac{\partial}{\partial t}\left(\mathrm{tr}\,\underline{\underline{\varepsilon}}\right) = \frac{1}{T_c}\left(\frac{\partial}{\partial\theta}\,\mathrm{tr}\,\underline{\underline{\varepsilon}} + \frac{1}{\alpha}\frac{\partial}{\partial\tau}\,\mathrm{tr}\,\underline{\underline{\varepsilon}}\right) \tag{B.3}$$

which can be written after development in Taylor's series:

$$\frac{\partial}{\partial t}\left(\mathrm{tr}\,\underline{\underline{\varepsilon}}\right) = \frac{1}{T_c}\left\{\frac{1}{\alpha}\frac{\partial}{\partial\tau}\left(\mathrm{tr}\,\underline{\underline{\varepsilon}}^{(0)}\right) + \frac{\partial}{\partial\tau}\,\mathrm{tr}\,\underline{\underline{\varepsilon}}^{(1)} + \frac{\partial}{\partial\theta}\,\mathrm{tr}\,\underline{\underline{\varepsilon}}^{(0)} + \dots\right\} \tag{B.4}$$

Inserting (B.2) and (B.4) in (B.1) and equating the terms on both sides at the same order in α, it comes:

(i) to the order –1:

$$\frac{\partial}{\partial\tau}\left(\mathrm{tr}\,\underline{\underline{\varepsilon}}^{(0)}\right) = 0 \tag{B.5}$$

(ii) to the order 0 :

$$\frac{\partial}{\partial\tau}\left(\mathrm{tr}\,\underline{\underline{\varepsilon}}^{(1)}\right) + \frac{\partial}{\partial\theta}\left(\mathrm{tr}\,\underline{\underline{\varepsilon}}^{(0)}\right) = \Delta'\left(p'^{(0)}\right) \tag{B.6}$$

The diffusion equation takes therefore two different forms, depending on the time scale considered.

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Figures caption

Fig. 1 Problem definition.

Fig. 2 Pore pressure variations during cyclic loading.

Fig. 3 Volume compressibility of coarse-grained materials (after Banerjee et *al).*

Fig. 4 Dynamic pore pressure increase in coarse-grained materials (after Banerjee et *al).*

Fig. 5 Pore pressure buildup in a cyclic triaxial test:
(a) stress ratio = 0.4 stress ratio = 0.6

Fig. 6 Foundation cross section of a bridge pier.

Fig. 7 Fourier amplitude of the shear force at the foundation level.

Fig. 8 Finite element model.

Fig. 9 Contours of shear stress D in the gravel layer.

Fig. 10 Time history of pore pressure buildup; foundation edge.

Fig. 11 Time history of pore pressure buildup; 13m from the center.

Fig. 12 Time history of pore pressure buildup; foundation center.